Friction-based scaling of streamwise turbulence intensity in zero-pressure-gradient and pipe flows

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Abstract

We explore the analogy between asymptotic scaling of two canonical wall-bounded turbulent flows, i.e. zero-pressure-gradient and pipe flows; we find that these flows can be characterised using similar scaling laws which relate streamwise turbulence intensity and friction.

1. Introduction

A recent paper \cite{1} on zero-pressure-gradient (ZPG) flow has introduced an asymptotic scaling law:

\[ \tilde{U}_\tau \sim \frac{1}{\sqrt{\tilde{\delta}}} \]

(1)

where

\[ \tilde{U}_\tau = \frac{U_\tau \nu}{M} \sim \frac{\nu}{U_\tau \delta} = \frac{1}{Re_\tau} \]

(2)

is named the ‘dimensionless drag’ and

\[ \tilde{\delta} = \frac{\delta M}{\nu^2} \sim \frac{\delta^2 U_\tau^2}{\nu^2} = Re_\tau^2 \]

(3)

is the friction Reynolds number squared. Here, \( U_\tau \) is the friction velocity, \( M \) is the kinematic momentum rate through the boundary layer, \( \nu \) is the kinematic

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viscosity and $\delta$ is the boundary layer thickness. Note the asymptotic scaling $M \sim U^2 \delta$ [1].

In this paper we will show that Equation (1) is equivalent to a corresponding equation for pipe flow [2, 3]. As a consequence, streamwise turbulence intensity scales in a similar way with drag for both ZPG and pipe flows.

Our study employs global averaging; similarities between local streamwise turbulence intensity for the same canonical flows have been treated in e.g. [4, 5].

The paper is organized as follows: In Section 2 we briefly review results from asymptotic scaling in pipe flows; these findings are related to ZPG flows in Section 3 and we conclude in Section 4.

2. Asymptotic pipe flow scaling

The material in this section is a summary of research contained in [2, 3].

For pipe flow, the streamwise turbulence intensity $I_{\text{pipe}}$ scales with the ratio of the friction and mean velocities:

$$I_{\text{pipe}} \sim \frac{U_\tau}{U_m} \sim \frac{Re_\tau}{Re_D},$$

where $U_m$ is the mean velocity and $Re_D = DU_m/\nu$ is the Reynolds number based on the pipe diameter $D$. The pipe friction factor $\lambda_{\text{pipe}}$ scales with the square of this ratio:

$$\lambda_{\text{pipe}} \sim \frac{U^2_\tau}{U^2_m} \sim \frac{Re^2_\tau}{Re^2_D}$$

As a consequence, the streamwise turbulence intensity scales with the square root of the friction factor:

$$I_{\text{pipe}} \sim \sqrt{\lambda_{\text{pipe}}}$$

We note the structural similarity between Equations (1) and (6). An example of the scaling using Princeton Superpipe measurements [6, 7] is Equation (23) in [3]:

$$I_{\text{pipe area, AM}} = 0.6577 \times \lambda_{\text{pipe}}^{0.5531},$$

where AM is an abbreviation for arithmetic mean.
3. Equivalence between zero-pressure-gradient and pipe flows

For ZPG flows, we can define a Reynolds number based on the free-stream velocity $U_\infty$ outside the boundary layer:

$$Re_\delta = \frac{\delta U_\infty}{\nu},$$  \hspace{1cm} (8)

which is equivalent to $Re_D$ for pipe flow. This Reynolds number can be used to define the ZPG streamwise turbulence intensity:

$$I_{ZPG} \sim \frac{Re_\tau}{Re_\delta} \sim \frac{1}{U_\tau Re_\delta}$$ \hspace{1cm} (9)

and the ZPG friction factor:

$$\lambda_{ZPG} \sim \frac{Re_\tau^2}{Re_\delta^2} \sim \frac{\tilde{\delta}}{Re_\delta^2}$$ \hspace{1cm} (10)

We now apply Equation (6) to ZPG flow:

$$I_{ZPG} \sim \sqrt{\lambda_{ZPG}}$$ \hspace{1cm} (11)

Rewriting Equation (11) using Equations (9) and (10) we have:

$$\frac{1}{U_\tau Re_\delta} \sim \frac{\sqrt{\delta}}{Re_\delta}$$ \hspace{1cm} (12)

or:

$$\frac{1}{U_\tau} \sim \sqrt{\tilde{\delta}}$$ \hspace{1cm} (13)

Comparing Equations (11) and (13) we identify the following analogies:

$$\tilde{\delta} \sim \tilde{\lambda}_{ZPG}$$ \hspace{1cm} (14)

$$\tilde{U}_\tau \sim \frac{1}{I_{ZPG}}$$ \hspace{1cm} (15)
We note that Equations (7) and (8) in [1] - the 'discrete' model - can be inverted and written as:

\[
\tilde{I}_{ZPG} \sim \frac{1}{\tilde{U}_r} = 6.6033 \times \tilde{\delta}^{0.55745}, \quad 10^8 \leq \tilde{\delta} \leq 10^{10}
\] (16)

\[
\tilde{I}_{ZPG} \sim \frac{1}{\tilde{U}_r} = 9.2005 \times \tilde{\delta}^{0.54261}, \quad 10^{10} \leq \tilde{\delta} \leq 10^{12}
\] (17)

where the exponents are very close to the ones typically found for pipe flow, see Equation (7) and [3]. A 'continuous' model is also introduced in [1] which covers the entire range of \( \tilde{\delta} \).

4. Conclusions

We have explored the correspondence between zero-pressure-gradient (ZPG) and pipe flows for asymptotic scaling of streamwise turbulence intensity with friction. It is demonstrated that similar scalings are valid for both types of flows; the 'dimensionless drag' for ZPG flow is equivalent to the inverse of the streamwise turbulence intensity.

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References


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