MATHMET 2025

Modelling of multiphase flow measurements in Coriolis flowmeters:

Status, cross-disciplinary applications and future developments

RI.
SE

"A formula is worth a thousand numerical simulations!"

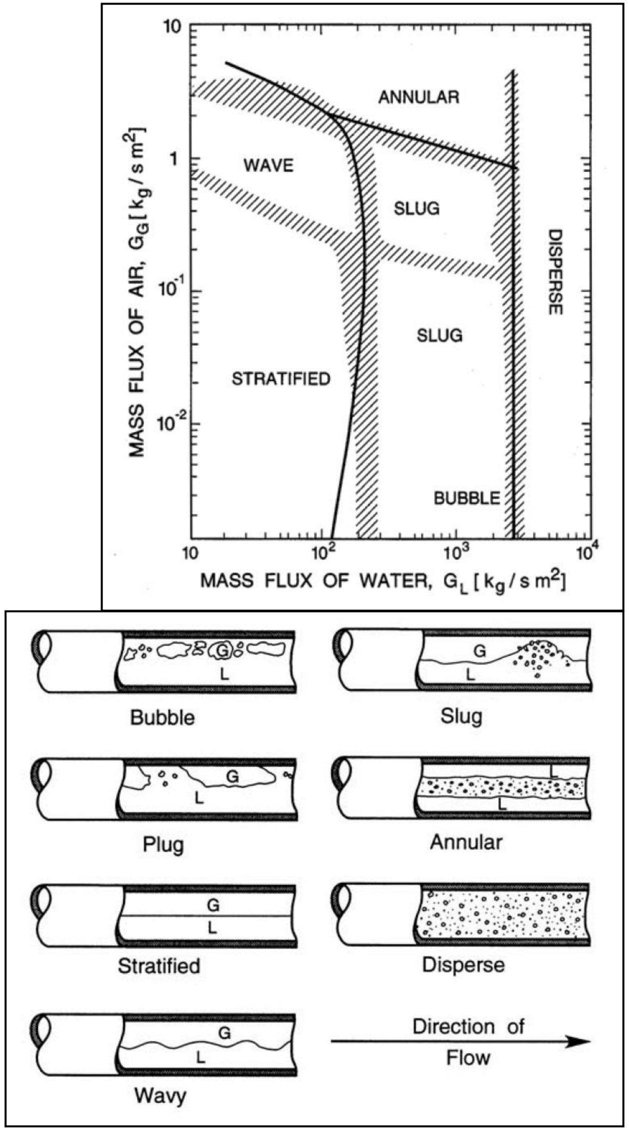
Michael Howe

"Since all models are wrong the scientist must be alert to what is importantly wrong."

George Box

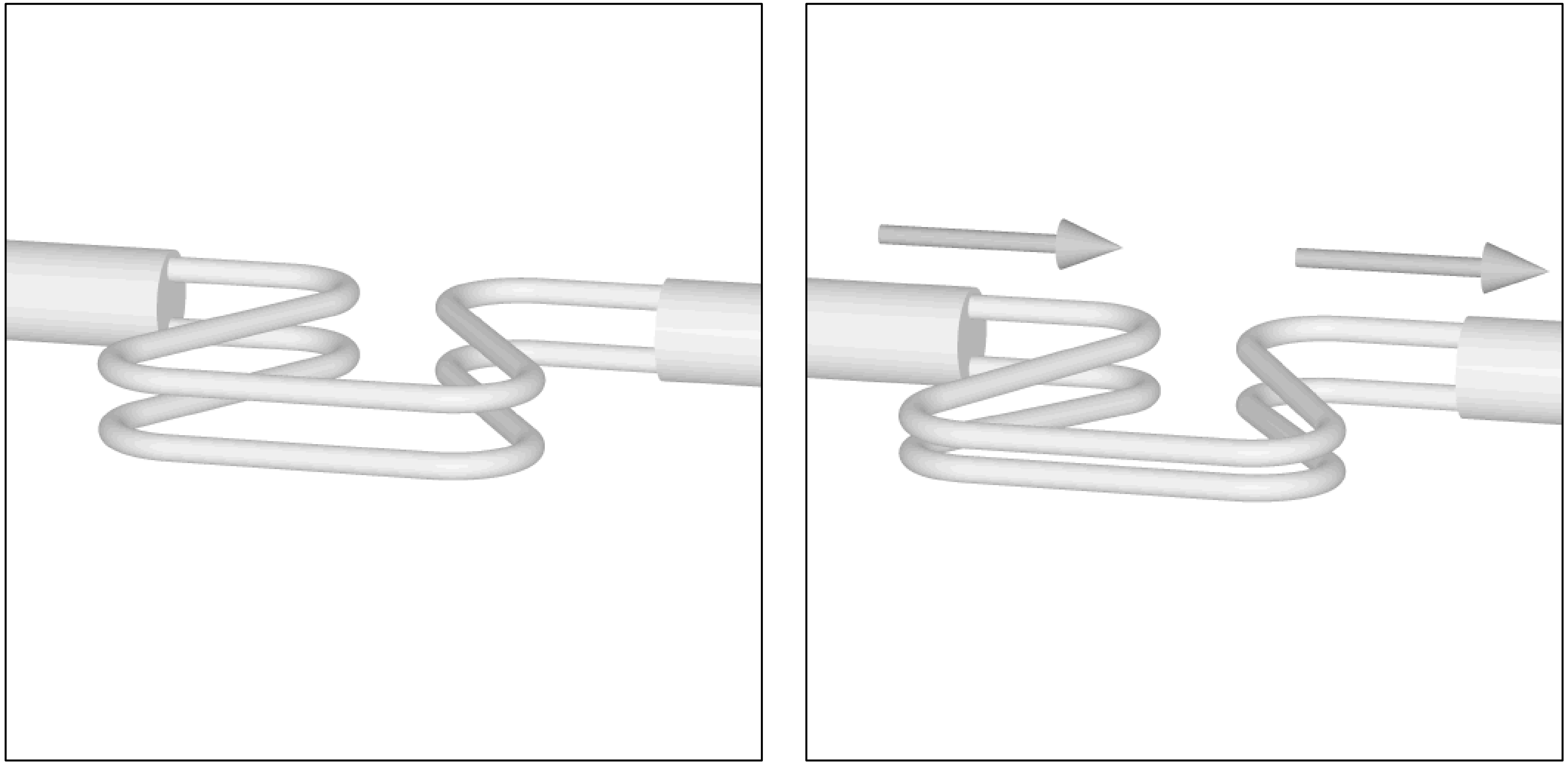
Outline

1. Coriolis flowmeters and two-phase flow
2. Important physical phenomena
3. The ”bubble theory”→”bubble model”:
    A. Physical picture
    B. Cross-disciplinary applications
4. Brief state-of-the-art review
5. Future developments



[Weisman 1983]

Coriolis flowmeters



Two-phase flow

402 M. Henry et al. / Flow Measurement and Instrumentation 17 (2006) 399–413

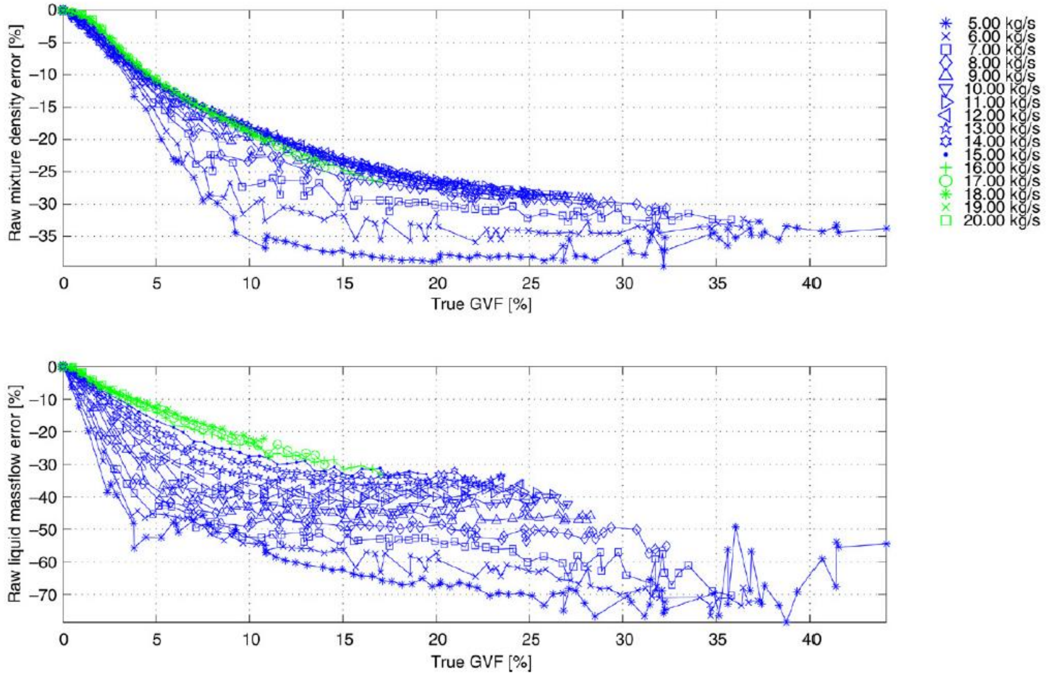


Fig. 2. Raw massflow and density errors for a 75 mm Coriolis meter in horizontal alignment under two-phase flow; the liquid is water and the gas is air.

Important physical phenomena

- Two physical phenomena:
  - Bubble theory (or phase decoupling) [Hemp et al. 1989-2003]
  - Compressibility [Hemp and Kutin 2006]

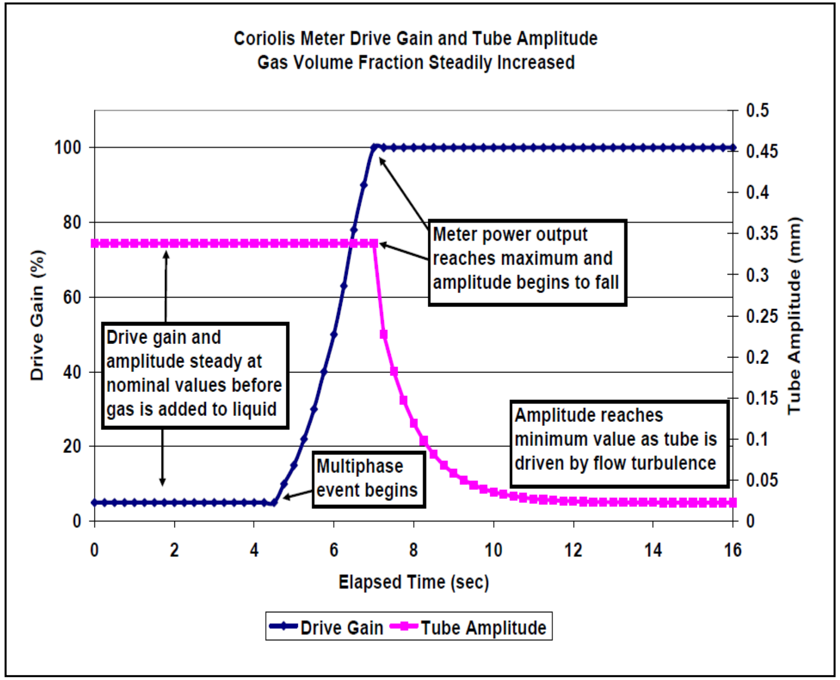
Table 1
Overview of measurement errors.

Phase	Compressibility error	Phase decoupling error
Liquid	Small	Not applicable
Gas	Medium	Not applicable
Two-phase (e.g., gas and liquid)	Large	Large

[Basse 2014a]

Bubble theory

- Important consequences:
  - Measurement errors
  - Damping

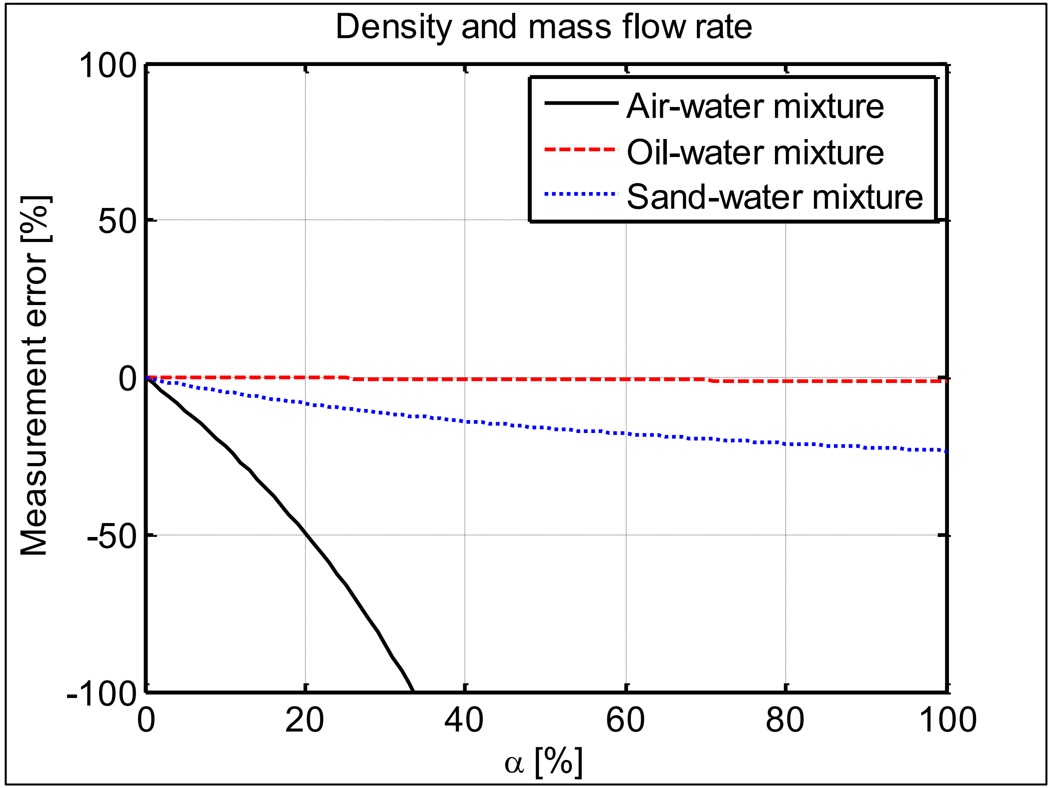


[Weinstein 2008]

- In this presentation we focus on an air-water mixture:
  - Air entrained in water
  - Gas-void fraction (GVF) is also called α

Bubble theory

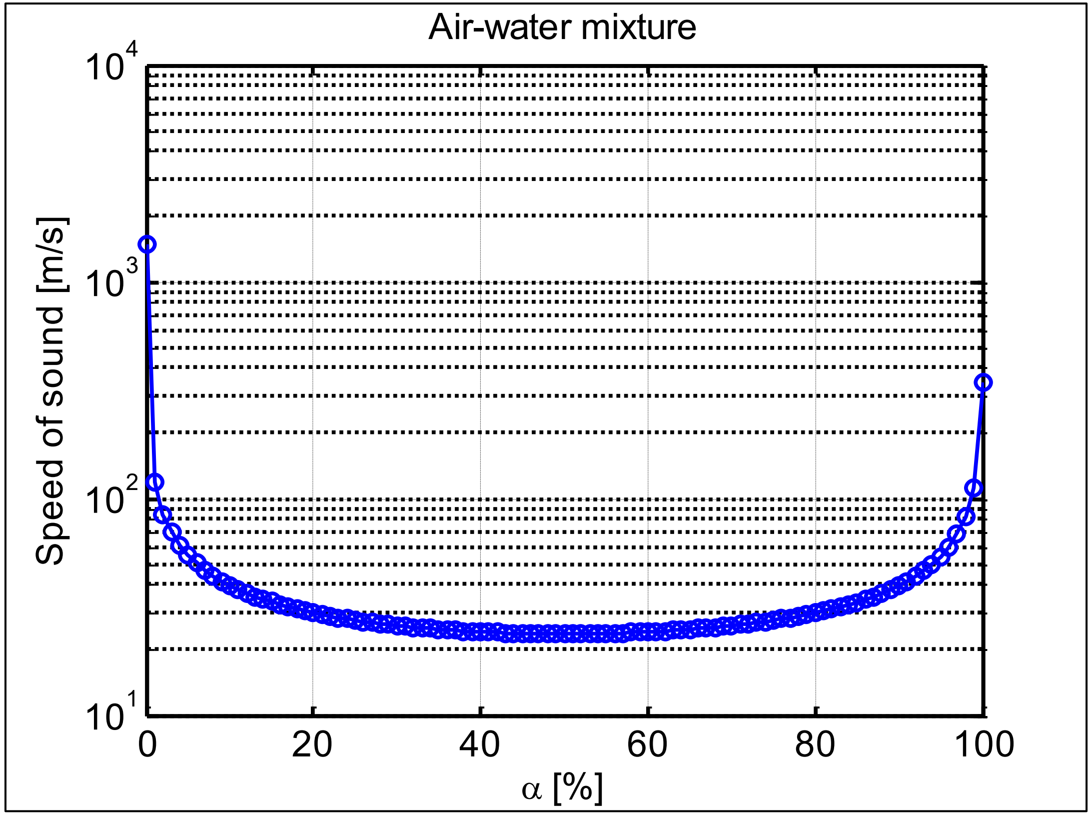
- Force on fluid:
  - $F_{f,z} = (\rho_f - \rho_p) V_p a_c F$
- Reaction force coefficient F:
  - Density, dynamic viscosity, Stk
- Total inertia reaction force on pipe:
  - $F_m = -\rho_f V_{f-p} a_c + F_{f,z}$
- Mass and density flow rate error:
  - $E_{\dot{m}} = E_d = \frac{\dot{m}_a - \dot{m}_{f-p}}{\dot{m}_{f-p}} = \frac{\alpha(\rho_f - \rho_p)(1-F)}{\alpha\rho_p + (1-\alpha)\rho_f}$



[Basse 2014a]

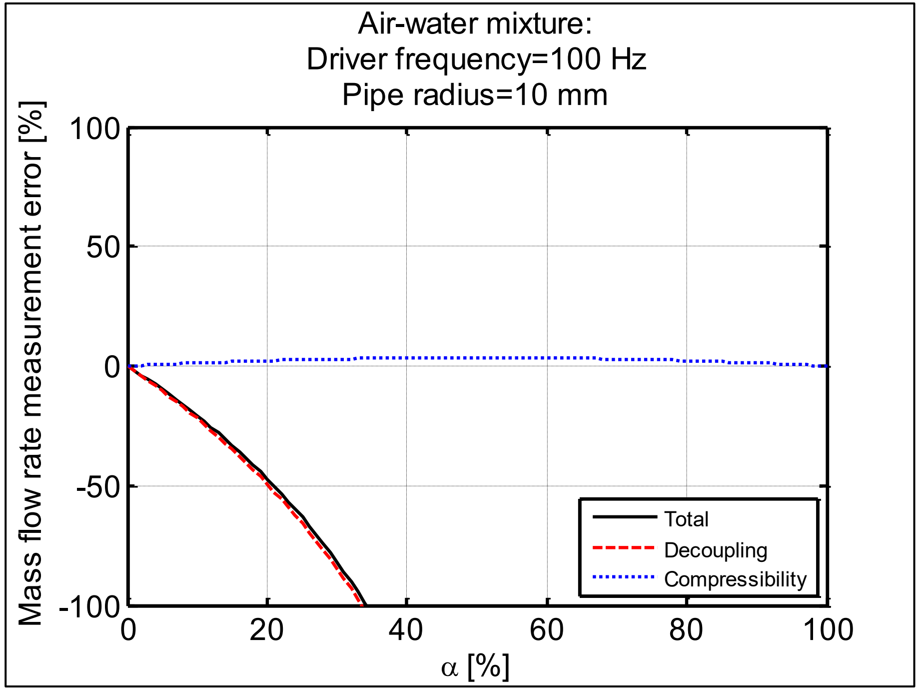
Compressibility

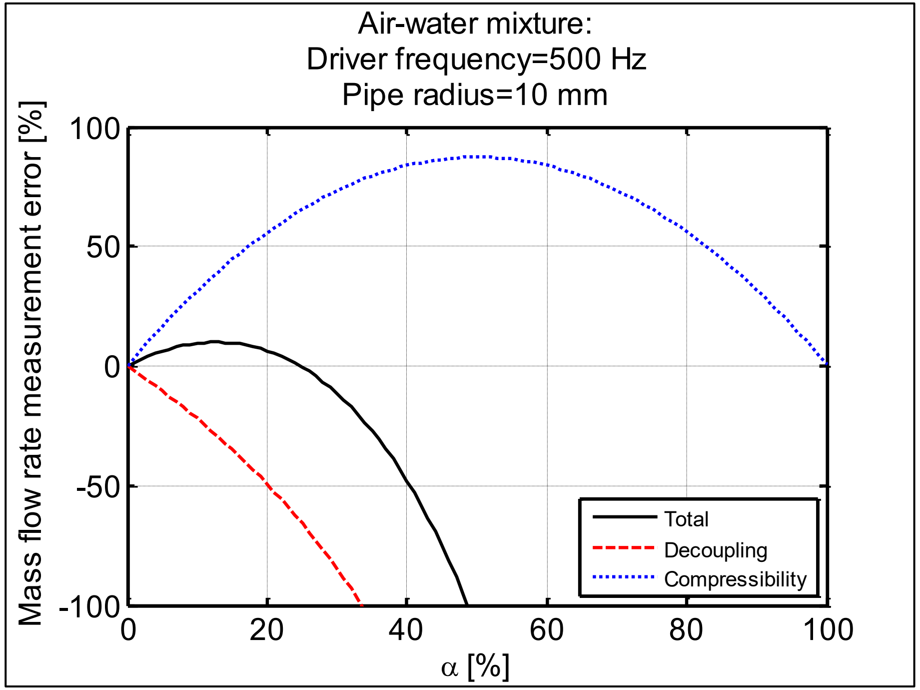
- Mixture speed of sound:
  - Minimum 25 m/s
  - Air 343 m/s
  - Water 1482 m/s



[Wood 1930]

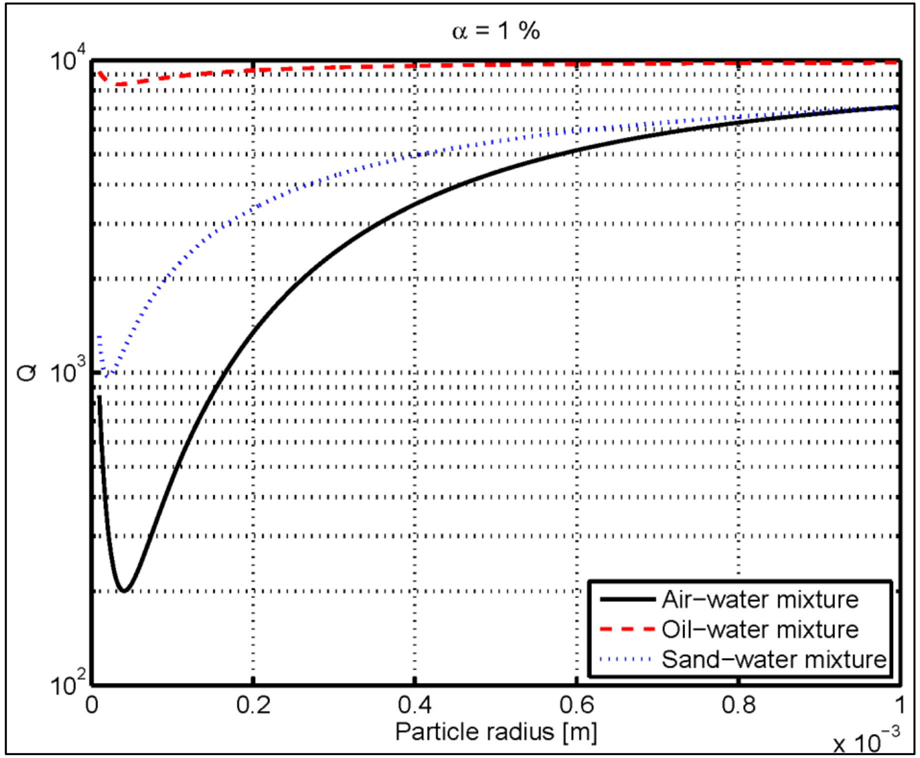
Measurement error: Air-water mixture

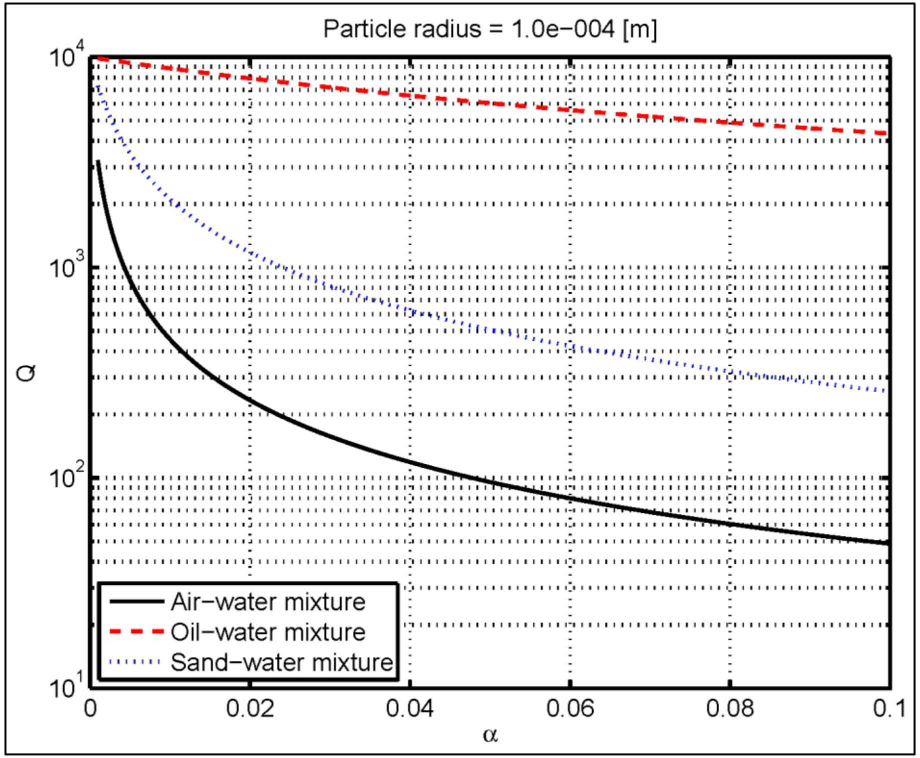




[Basse 2014a]

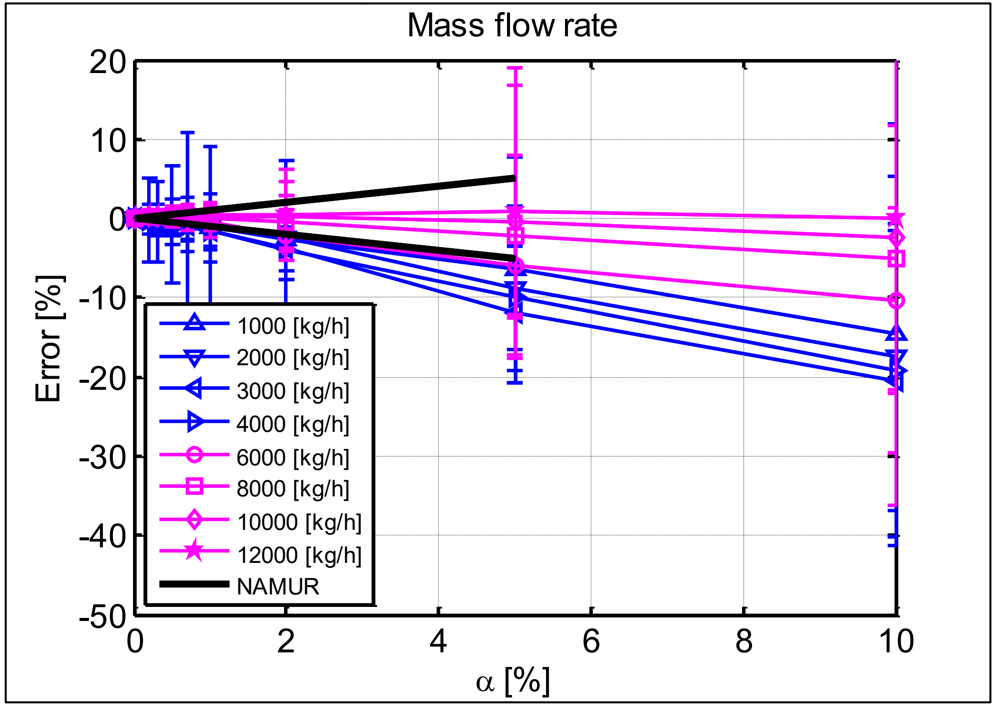
Damping: Three mixtures

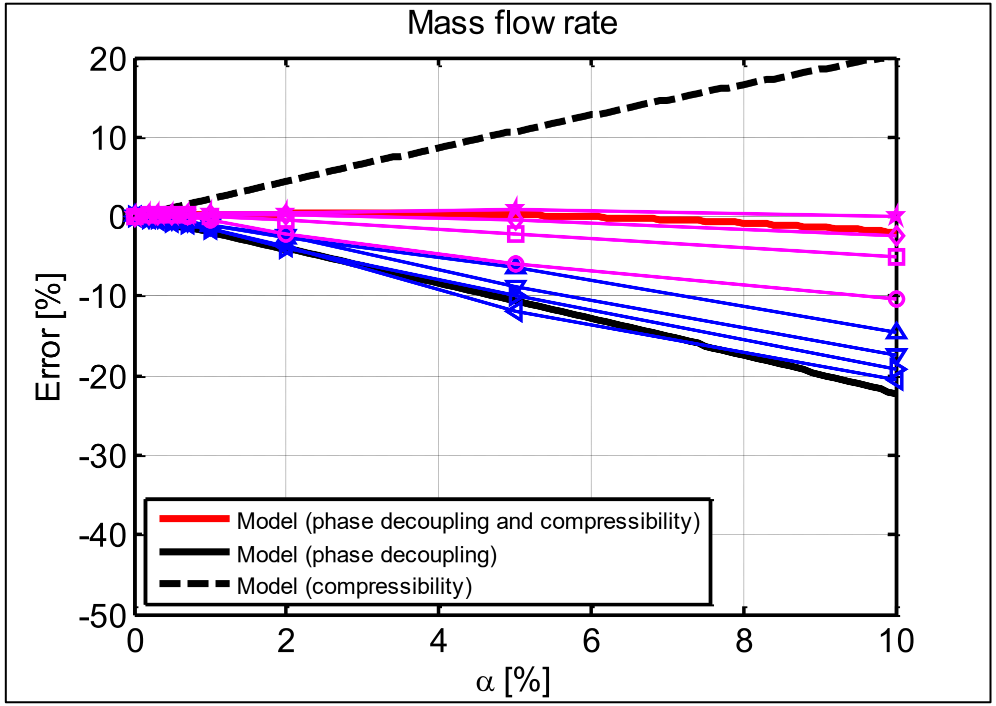




[Basse 2016]

Comparison to measurements





[Basse 2014b]

The "bubble theory": Cross-disciplinary applications

- Analogy of bubble theory with the bias flow aperture theory [Howe 1979]:
  - Single-phase bias flow through an aperture can be described as two-phase flow
  - Analogy to large eddy simulations (LES):
    - Large structures and small-scale turbulence
- Mean flow is a prerequisite for both physical effects, wake analogy

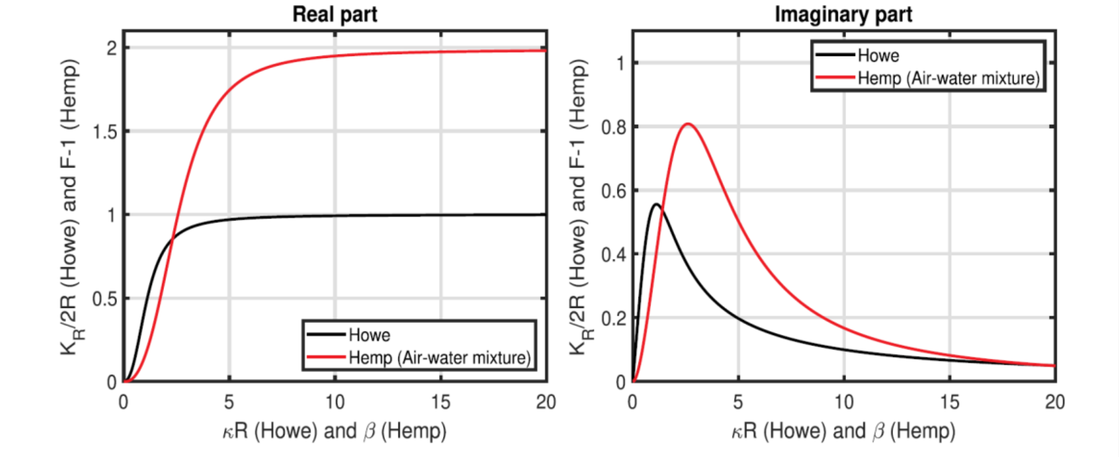


Fig. 3. Normalised Rayleigh conductivity and $F-1$ for an air-water mixture, left: Real part, right: Imaginary part.

[Basse 2020]

The "bubble theory": Cross-disciplinary applications

- Amended bubble theory applied to sound propagation and damping in aerosols and hydrosols:
  - [Temkin 1996]: Solid particle (SP) theory
  - [Hemp 2003]: Viscous particle (VP) theory:
    - Bubble theory amended with [Galindo and Gerbeth 1993]
- Differences in sound attenuation for air bubbles entrained in water

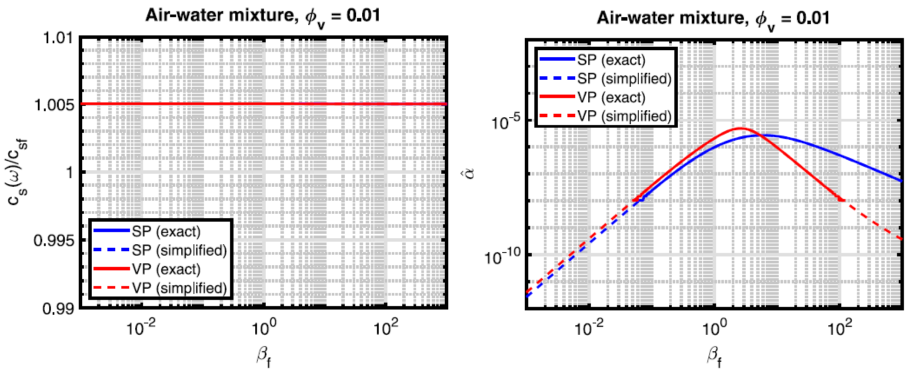


Fig. 6. Air–water mixture, left: Normalized sound speed, right: Nondimensional attenuation. Exact expressions are marked by solid lines and simplified expressions are marked by dashed lines.

[Basse 2021]

Brief state-of-the-art review

1. [Hemp et al. 1989-2003]: Bubble theory:
   - [Yang and Leal 1991]: Force on accelerating spherical drop
2. [Liu et al. 2001]: First application of ANNs (MLP/RBF)
   - [Henry et al. 2006, 2013]: Application to two- and three-phase flow
3. [Hemp and Kutin 2006]: Compressibility
4. [Gysling 2007]: Aeroelastic model
5. [Weinstein 2008]: Particle motion

Brief state-of-the-art review

6. [Tzu 2008]: Moving resonator model
7. [Basse 2014-2016]: Review of the bubble theory (errors and damping)
8. [Wang et al. 2017]: Application of ANNs (BP/RBF), SVM and GP (SVM best)
9. [Yan et al. 2018]: Soft computing review:
   – Machine learning (ANN/SVM/DL), evolutionary computation (GA/GP), fuzzy logic, probabilistic reasoning (Bayesian network), hybrid models
10. [Li et al. 2019]: Signal processing (filtering), reduced standard deviation

Brief state-of-the-art review

11. [Meribout et al. 2020]:
    – ANN (MLP) and bubble theory (no compressibility) combined with separator
12. [Ibryaeva et al. 2021]: Linear and non-linear SVMs (linear best)
13. [Gysling 2023]: Aeroelastic model combined with SoS measurement
14. [Bahrami et al. 2024]: Review of ANNs for flow regime, GVF and flow rate
15. [Wernli et al. 2025]: Particle motion, changing GVF along flow

Future developments

- Combine and improve:
  - Modelling and simulation
  - Augmented measurements
  - Soft computing
- PINNs:
  - [Raissi et al. 2017; Li and Bai 2023: PGNN for DP meter]
- Open measurement databases [Ibryaeva et al. 2020]
- Uncertainty quantification

Table 4
Bubble theory assumptions.

Part of system	Assumptions
Overall	• The effect of gravity is neglected
Flow (particles and fluid)	• Particles and fluid move at the same velocity • Particles homogeneously dispersed in fluid • Plug flow • Incompressible
Particles	• Sphere • Surface tension is not taken into account • Single radius • Non-interacting
Container	• Rigid (no fluid-structure interaction) • Oscillation amplitude is small compared to the particle radius • Oscillation frequency: Fast compared to the flow speed Independent of the volumetric particle fraction
Particles and container	• The particle is far from wall of the container (the particle size is small relative to the size of the container)

[Basse 2014a]

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